

References for the Colloquium Talk

“OPTIMAL PACKINGS: PROBLEMS FOR THE AGES”

presented by

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at the Aspen Center for Physics, June 29, 2006

Abstract

Packing problems, such as how densely solid objects occupy space, have fascinated people since the dawn of civilization, and continue to intrigue scientists because of their connection to a host of problems that arise in the physical sciences, mathematics and biology. While optimal packing problems are intimately related to ground states of condensed matter, disordered “jammed” sphere packings have been employed to model the glassy state of matter. Sphere packings in high dimensions have relevance in communications theory. Discrete geometers have a longstanding interest in packing problems. I will review some of the history of packing problems, discuss some recent developments, and briefly mention open problems.

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